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Neutral color-spin-locking phase in neutron stars

D.N. Aguilera[a]

Department of Applied Physics, University of Alicante, Apartado Correos 99, 03080 Alicante, Spain

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Abstract. We present results for the spin-1 color-spin-locking (CSL) phase using a NJL-type model in two-flavor quark matter for compact stars applications. The CSL condensate is flavor symmetric and therefore charge and color neutrality can easily be satisfied. We find small energy gaps $\simeq 1\,\mathrm{MeV}$, which make the CSL matter composition and the EoS not very different from the normal quark matter phase. We keep finite quark masses in our calculations and obtain no gapless modes that could have strong consequences in the late cooling of neutron stars. Finally, we show that the region of the phase diagram relevant for neutron star cores, when asymmetric flavor pairing is suppressed, could be covered by the CSL phase.

PACS. 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes – 26.60.+c Nuclear matter aspects of neutron stars

1 Introduction

The investigation of cold dense quark matter has received special attention due to the possible consequences for compact stars [1]. In particular, color superconducting quark matter phases enforcing color and charge neutrality has been widely studied [2]. Model calculations have shown that the intermediate density region of the neutral QCD phase diagram, where the quark chemical potential is not sufficiently large to have the strange quark deconfined, might be dominated by u, d quarks [3]. If this is the case, two-flavor quark matter phases may occupy a large volume in the core of compact stars [4].

On the other hand, local charge neutrality disfavors the occurrence of phases with large gaps where quarks with different flavor pair in a spin-0 condensate, such as the 2SC phase [5], in which quarks pair in *e.g.* $(u_r d_g)$ and $(d_r u_g)$ diquarks leaving the u_b, d_b unpaired. Therefore, while the occurrence of neutral 2SC pure phase is rather model dependent and might be unlikely for moderate coupling constants [6], other phases such *e.g.* with spin-1 pairings [7], could be relevant for neutron star phenomenology. Specially, because these condensates with small energy gaps ($\Delta \simeq 1\,\mathrm{MeV}$) do not influence the equation of state (EoS) (it is not distinguishable from the normal-quark (NQ) matter EoS) but they strongly affect the transport and thermal properties of quark matter [8] and consequently the neutron star cooling. The unpaired quarks in the core lead to rapid cooling via the direct Urca process, incompatible with the observations. Phases that present no gapless modes prevent the direct Urca to work uncontrolled suppressing the neutrino emissivities and could explain the observed data [9].

We consider in this work the color-antitriplet single-flavor spin-1 pairing in the color-spin-locking (CSL) phase [7] and compare it with the 2SC phase. Our results for the CSL phase are obtained using the Nambu–Jona-Lasinio (NJL) model [10] keeping finite quark masses and thus obtaining no gapless modes. Since these condensates are color neutral and single-flavor, neutral beta-equilibrated CSL quark matter is obtained easily. We present also the thermal behavior of the neutral CSL phase showing the phase diagram. Finally, we stress important features of the CSL phase that could give a consistent picture of a compact star with a superconducting quark matter core.

2 Flavor symmetric (CSL) vs. flavor asymmetric (2SC) pairing

We consider two-flavor ($f = u, d$) quark matter, assuming that the strange-quark mass is large enough to appear only at higher densities. In the NJL-type[1] models the quarks

[1] We consider also nonlocal extensions of the NJL model: the quark interactions act over a certain range introducing momentum-dependent form factors in the the current-current interaction terms. The inclusion of high-momenta states beyond the NJL cutoff reduces the diquark condensates and lowers the density for the chiral phase transition (for a discussion see [11]).

[a] e-mail: deborah.aguilera@ua.es; supported by VESF-Fellowships EGO-DIR-112/2005.

Table 1. Two-flavor quark matter phases: flavor asymmetric spin-0 2SC and flavor symmetric spin-1 CSL. For $[S \ldots A]$ the following pairs of indices apply: $S, A = 3,2; 1,7; 2,5$.

Phase	Condensate Δ	Diquarks	Free
2SC (spin-0)	$2G_1\langle\, \psi^T\ C\ \gamma_5\ \tau_2\ \lambda_2\ \psi\,\rangle$	$u_r d_g,\ d_r u_g$	$u_b,\ d_b$
CSL (spin-1)	$4H_v\langle\, \psi^T\ C\ \gamma_{[S}\ \lambda_{A]}\ \psi\,\rangle$	$u_r u_g,\ u_g u_b,$ $u_b u_r,\ (u \to d)$	–

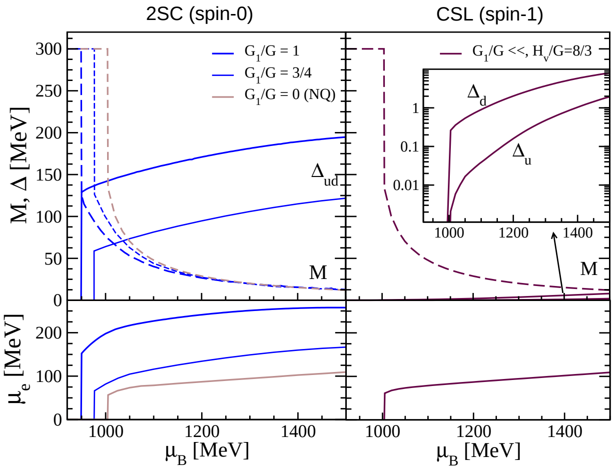

Fig. 1. Gap equations solutions for neutral matter in the flavor asymmetric spin-0 2SC phase (on the left) and in the flavor symmetric spin-1 CSL phase (on the right) at $T = 0$. The inset shows the small CSL gaps $\simeq 0.01$–1 MeV. Upper panel: dynamical mass M and energy gap Δ as a function of the baryon chemical potential μ_B. Lower panel: electron chemical potential μ_e *vs.* μ_B. Different 2SC couplings are considered from $G_1/G = 1$ (dominant 2SC, see fig. 3) down to $G_1/G = 0$ ($\Delta \equiv 0$, NQ matter). The CSL coupling is taken as $H_v/G = 8/3$.

interact locally by a 4-point vertex effective force. The NJL Lagrangian $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{q\bar{q}} + \mathcal{L}_{qq}$ contains a free part $\mathcal{L}_0$, a quark-antiquark channel $\mathcal{L}_{q\bar{q}}$ that causes spontaneous chiral symmetry breaking with condensates $\sigma = \langle \bar{q}q \rangle$ and a diquark channel $\mathcal{L}_{qq}$ that describes color superconductivity with condensate Δ. The constituent-quark mass is defined as $M = m - 4G\sigma$ and the energy gaps Δ are listed in table 1 for the phases: spin-0 2SC and spin-1 CSL.

Linearizing $\mathcal{L}_{\text{eff}}$ in the presence of the condensates the thermodynamical potential $\Omega(T, \{\mu_f\})$ can be derived. For the quark sector, in the 2SC case we obtain

$$\Omega_q = 4G\sigma^2 + \frac{|\Delta|^2}{4G_1} - 2\sum_{\pm, j=1}^{3} \int \left[E_j^{\pm} + 2T \ln\left(1 + e^{-E_j^{\pm}/T}\right) \right]$$

and for CSL, since the flavors decouple, $\Omega_q = \sum_f \Omega_f(T, \mu_f)$

$$\Omega_f = 4G\sigma^2 + \frac{3|\Delta_f|^2}{8H_v} - \sum_{k=1}^{6} \int \left[E_k^f + 2T \ln\left(1 + e^{-E_k^f/T}\right) \right],$$

where $E_j^{\pm}$ and E_k^f are the corresponding dispersion relations and the integration is in the momentum space.

The lepton sector is modeled as an ideal electron gas with chemical potential μ_e. At the mean-field level, the stationary points $\delta\Omega/\delta\Delta = \delta\Omega/\delta M = 0$ define a set of gap equations for Δ and M. The stable solution is the one which corresponds to the absolute minimum of Ω.

The parameters (quark mass $m = 5$ MeV, the coupling $G = 4.66\,\text{GeV}^{-2}$ and the cutoff $\Lambda = 664$ MeV) are chosen to fit the pion mass and the decay constant to a vacuum constituent-quark mass equals 300 MeV.

3 Results and discussion

We solve self-consistently the gap equations for the dynamical mass M and the energy gap Δ for beta-equilibrated neutral matter: our solutions satisfy that the total quark electric charge $\mu_Q = -\mu_e$ and that the total charge density $n_Q - n_e = \frac{2}{3}n_u - \frac{1}{3}n_d - n_e$ vanishes. In fig. 1 we show M and Δ as a function of the baryon chemical potential μ_B for the flavor asymmetric spin-0 2SC phase on the left and for the flavor symmetric spin-1 CSL phase on the right. While, for a fixed μ_B, M presents a similar magnitude for both phases, Δ differs by order of magnitudes: $\simeq 100$ MeV for 2SC, $\simeq 1$ MeV for CSL. Moreover, the strength of the coupling constant determines whether the 2SC phase occurs: for *strong coupling* $G_1/G = 1$, it presents large gaps $\Delta \simeq 150$–200 MeV that decrease as soon as the coupling does ($\Delta \simeq 50$–100 MeV for the usual Fierz value $G_1/G = 3/4$) and vanishing for values lower than a critical one. The crucial point is that the coupling should be large enough to pair quarks of different flavors overcoming a large Fermi sea mismatch ($\simeq 60$–80 MeV). As a consequence, asymmetric flavor pairing might not be favorable unless the coupling is very strong (at least larger than $G_1/G = 3/4$)[2]. Therefore, flavor symmetric pairing becomes important when G_1/G is not large enough to have 2SC superconductivity.

On the other hand, the occurrence of the flavor symmetric CSL pairing is not affected by the charge neutrality constraint. The CSL condensates, having small energy gaps in comparison to the free energy of the system, do not modify the thermodynamic properties respect to the normal phase. In fig. 2 we show the number density for quarks, n_u, n_d, and for electrons, n_e, as a function of μ_B for the 2SC (on the left) and for the CSL phase (on the right). Different values of the coupling for the 2SC phase from $G_1/G = 1$ down to $G_1/G = 0$ (NQ) are considered. We clearly see that $n_i^{2\text{SC}, G_1/G=0} = n_i^{\text{NQ}} \simeq n_i^{\text{CSL}}$ for each particle specie i, so the two phases, CSL and NQ, are indistinguishable from the matter composition. This conclusion holds also for other thermodynamic quantities like pressure or energy density and therefore for the EoS.

[2] Actually, the critical value of G_1/G at which the 2SC condensate breaks down is model and parameterization dependent. NJL calculations have obtained no pure 2SC at intermediate densities ($\mu_B = 1200$ MeV) below $G_1/G = 3/4$, see [3,6].

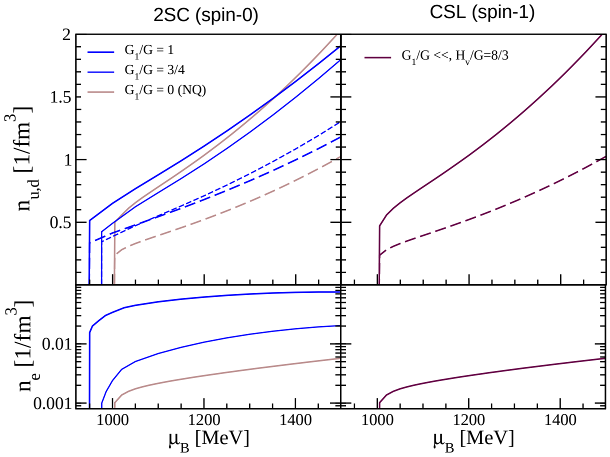

Fig. 2. Quark (d solid line, u dashed line) and electron number densities for 2SC (on the left) and CSL (on the right) as a function of μ_B for neutral matter at $T = 0$.

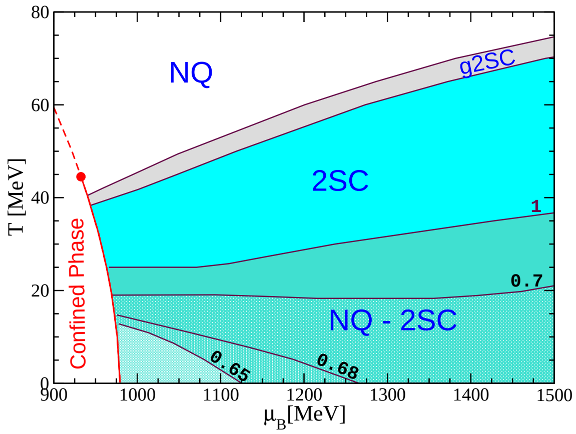

Fig. 3. Phase diagram for the intermediate density region relevant for neutron star cores. For *strong coupling* ($G_1/G \simeq 1$), flavor asymmetric spin-0 2SC phase is dominant. The mixed phase NQ-2SC assures global charge neutrality. The volume fraction of the 2SC sub-phase is indicated by numbers over the corresponding lines (Gaussian form factor [6]). As T increases, the volume fraction of the 2SC increases up to pure 2SC. Gapless 2SC is found before the transition to NQ occurs.

Finally, we found that while the phase diagram for *strong coupling* is dominated by the 2SC phase (fig. 3) for *intermediate or weak coupling*, the CSL phase is favorable (fig. 4). Note the low CSL critical temperatures $T_c^{\rm CSL} \simeq 5\,\text{MeV}$ in contrast to the 2SC case, for which $T_c^{\rm 2SC} \simeq 50\,\text{MeV}$. Thus, we expect that in the cooling of a neutron star, when the temperature has fallen below the MeV scale, a CSL superconducting quark core could develop. Stable hybrid-star configurations have been obtained with a relatively large NQ matter core [4], therefore, hybrid stars with a CSL superconducting core will be stable as well. Finally, a qualitative study of the interaction of the magnetic field with the CSL phase shows that a CSL core is consistent with recent observations and models of magnetized neutron stars [12].

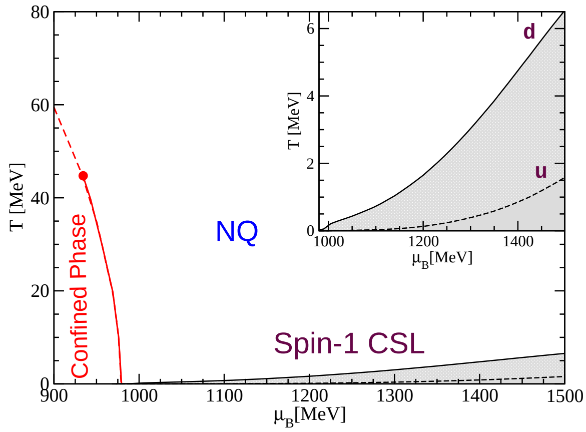

Fig. 4. Same as fig. 3 for *intermediate or weak coupling* ($G_1/G \leq 3/4$), for which flavor asymmetric pairing is no longer favorable. The volume fraction of the 2SC phase becomes very small ($< 10\%$) and the structure of fig. 3 disappears. Thus, the matter could be in the normal state (NQ) or in a phase with flavor symmetric pairing such as the spin-1 CSL phase.

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